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I. Solution by the **PROPOSER**, and **P. S. BERG**, Larimore, North Dakota.

Let x=the time.

Now the amount of \$200 for $(x-4)$ years—the amount of \$100 for $(x-2)$ years must=\$100.

$200+12(x-4)=152+12x$=amount of \$200 for $(x-4)$ years at 6%.

$100+6(x-2)=88+6x$=amount of \$100 for $(x-2)$ years at 6%.

$\therefore$ $(152+12x)-(88+6x)=100$. $\therefore$ $x=6$ years.

II. Solution by Hon. **J. H. DRUMMOND**, LL. D., Portland, Maine.

Computing at simple interest,

$\$\frac{200}{124}-\$\frac{100}{112}=\$72\frac{1}{217}$, amount equitably due now.

Hence, $\$100-\$72\frac{1}{217}=\$27\frac{216}{217}$ to be earned as interest.

$\$27\frac{216}{217}\div(72\frac{1}{217}\times\frac{6}{100})=5\frac{31}{75}$, the number of years required.

III. Solution by **NELSON S. RORAY**, South Jersey Institute, Bridgeton, New Jersey.

In this I assume interest to remain unpaid until the time of settlement, and to *draw no interest.*

A owes me to-day the present worth of \$100 due in 2 years at 6%, or \$89.29.

I owe A to-day the present worth of \$200 due in four years at 6%, or \$161.29.

That is, I owe A \$72 more than he owes me. Hence the problem reduces itself to the question, when will the excess of my interest over his plus \$72 amount to \$100? That is, my interest must exceed his by \$28.

My yearly excess is \$4.32. Hence to gain \$28, 6.481 years will be required.

GEOMETRY.

Conducted by **B. F. FINKEL**, Springfield, Mo. All contributions to this department should be sent to him.

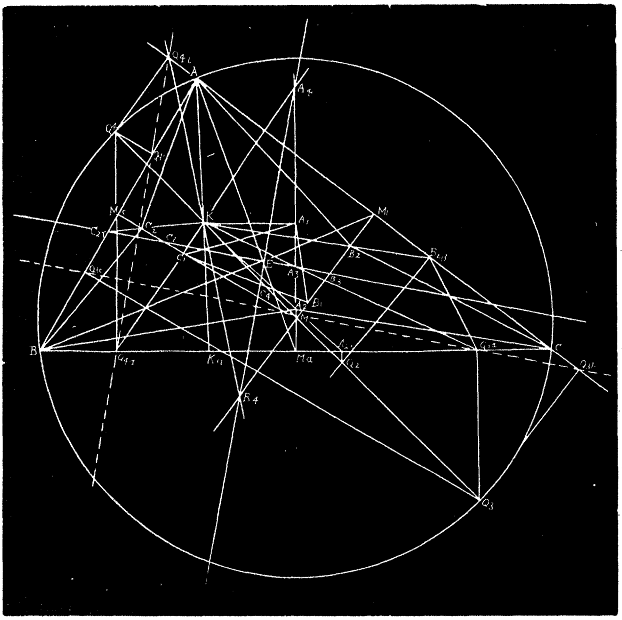
SOLUTIONS OF PROBLEMS.

65. Proposed by **I. J. SCHWATT**, Ph. D., Professsor of Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.

The axes of the ellipse isogonal to Lemoine's line with respect to a triangle (Steiner's ellipse), are parallel to Simson's lines belonging to the extremities of Brocard's Diameter.

Solution by F. M. McGAW, A. M., Professor of Mathematics, Bordentown Military Institute, Bordentown, New Jersey.

Let the triangle be ABC ; center of circumcircle, M; $A_1B_1C_1$, the Brocard triangle with vertices on parallels thro' Grebe's point and perpendiculars at mid-points of sides of ABC. It is known that ABC and $A_1B_1C_1$ are similar. Let E be the medio-centre, or centre of gravity, of ABC. It is known that E is also medio-centre of $A_1B_1C_1$. Let MK, Brocard's diameter (or the diameter of circle about Brocard's triangle) be produced to meet circumcircle of ABC in the points Q_3 and Q_4.



By the construction, the vertices of Brocard's triangle are also the vertices of three similar isosceles triangles ; for these isosceles triangles have as altitudes the perpendiculars from Grebe's point upon the sides of ABC, and it is known that these perpendiculars are as the sides. Hence the triangles have bases and altitudes proportional, and therefore are similar.

If, now, any three similar isosceles triangles be constructed upon the sides of ABC, their vertices A_2, B_2, C_2, will be the vertices of a triangle having the same medio-centre as ABC or $A_1B_1C_1$. The proof of this is similar to that which is known to establish E the same for ABC, and $A_1B_1C_1$.

Draw KA_2, KB_2, KC_2 to meet sides of triangle ABC in points $A_{2\alpha}$, $B_{2\beta}$, $C_{2\gamma}$ respectively. Then triangle $A_{2\alpha}B_{2\beta}C_{2\gamma}$ is similar to the triangle $A_2B_2C_2$, and centre of similitude is K. This may be proved as follows : Erect a perpendicular at $A_{2\alpha}$ to cut Brocard's diameter (produced) at Q_2, then

$$A_2M_a : KK_a = A_{2\alpha}A_2 : A_{2\alpha}K = Q_2M : Q_2K. \quad (1).$$

Triangles A_2BC, and B_2AC are similar by construction, hence

$$A_2M_a : B_2M_b = M_aC : M_bC = a : b = A_1M_a : B_1M_b,$$

where $a : b$ is ratio of two sides of triangle ABC.

We may write this last

$$A_2M_a : A_1M_a = B_2M_b : B_1M_b = B_{2\beta}B_2 : B_{2\beta}K = Q_2M : Q_2K \qquad (2).$$

From (1) and (2) we have,

$$A_{2\alpha}A_2 : A_{2\alpha}K = B_{2\beta}B_2 : B_{2\beta}K,$$

hence the lines A_2B_2 and $A_{2\alpha}B_{2\beta}$ become parallel, and if the same course of reasoning be pursued with regard to the other sides, the triangles are seen to be similar, with K the center of similitude.

Now, from the equality, $B_{2\beta}B_2 : B_{2\beta}K = Q_2M : Q_2K$, it follows that $B_{2\beta}Q_2$ is parallel to B_2M; and since B_2M is perpendicular to AC, $B_{2\beta}Q_2$ is also perpendicular to AC. Similarly, the perpendicular to AB at $C_{2\gamma}$ passes through Q_2, and we have already that the perpendicular to BC at $A_{2\alpha}$ passes through Q_2. If Q_2 be caused to coincide with either Q_3 or Q_4, then triangle $A_{2\alpha}B_{2\beta}C_{2\gamma}$ will degenerate into the straight lines $Q_{3a}Q_{3b}Q_{3c}$, and $Q_{4a}Q_{4b}Q_{4c}$ which are the Simson lines belonging to Q_3 and Q_4. Also triangle $A_2B_2C_2$ will degenerate into the straight lines $A_3B_3C_3$, and $A_4B_4C_4$, which are parallel to Simson's lines, to Q_3, Q_4. These lines will also pass through the medio-centre, E, since the triangles which degenerate continually have E as the medio-centre. Since the Simson lines are perpendicular to each other (see Geometry of Simson lines), these last mentioned lines through E, are perpendicular to each other. Since we know that the ellipse (Steiner's) has these lines for axes, the proposition is proved. Q. E. D.

NOTE. The above solution I got from Dr. Schwatt. An elegant demonstration of properties of the ellipse is given in Schwatt's Isogonal Curves, (Leach, Shewell & Sanborn, New York). F. M. M.

This problem was also solved by Prof. G. B. M. Zerr. Prof. William Hoover did not solve it, but referred to the proof given in *Casey's Analytical Geometry*, Edition of 1893, Articles 364, 365 (Cor. 1).

66. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

The locus of points whose polars with respect to a given parabola touch the circle of curvature at the vertex is an equilateral hyperbola.

I. Solution by the PROPOSER.

By Salmon's *Conic Sections*, Sixth Edition, Ex. 4, page 234, the equation to the circle osculating a parabola $y^2=px$......(1) at (x', y') is

$$(p^2+4px')(y^2-px)=\{2yy'-p(x+x')\}\{2yy'+px-3px'\}\ldots\ldots\ldots(2).$$